Ordinary Differential Equations
Exercise Sheet 3

Exercise 1. *Find the equilibrium points and examine their stability for the following equations:*

(i) $y' = 3y(1-y)$

(ii) $y' = y^2 - 6y - 16$

(iii) $y' = (y-2)(1-2\sin y)$

Exercise 2. *Compute the linearized equations in exercise 1 at every equilibrium point of the corresponding equation.*

Exercise 3. *Find the bifurcation values and draw the corresponding diagrams, showing the stability/instability of the equilibrium points in each case:*

(i) $y' = \mu y(1-y)^2 - y^3$

(ii) $y' = \mu y^2 + y^4$

for $\mu \in \mathbb{R}$

Exercise 4. *Compute the flow and integral curves for each of the following vector fields in* $\mathbb{R}^2$*:*

$$\text{(i) } \vec{v} = (-y_2, y_1), \qquad \text{(ii) } \vec{v} = (y_1, 2y_2), \qquad \text{(iii) } \vec{v} = (y_2, y_1),$$

where we recall that the flow of a vector field $\vec{\varphi}_t(\vec{\xi}) = \vec{\varphi}(t, \vec{\xi})$ *solves the IVP:*

$$\vec{y}\,' = \vec{v}(\vec{y}), \qquad \vec{y}(0) = \vec{\xi}, \qquad \vec{y} = (y_1, y_2),\ \vec{\xi} = (\xi_1, \xi_2).$$

Exercise 5. *Show that a necessary condition for the existence of a bifurcation for the family of equations* $y' = c + dy - y^3$ *is* $4d^3 = 27c^2$.

Exercise 6. *Show that the system*

$$\begin{cases} y_1' = (a - by_2)y_1 \\ y_2' = (dy_1 - c)y_2 \end{cases}$$

preserves the first quadrant, that is to say, for positive initial conditions $y_1(0), y_2(0) \geq 0$*, it holds* $y_1(t), y_2(t) \geq 0$ *for all* $t \geq 0$.